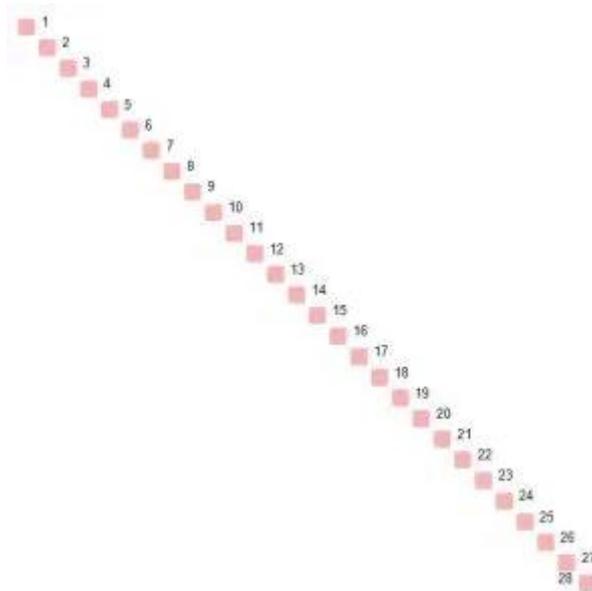# Complexity-Adjusted Standard Deviation

📅 October 27th , 2018

Standard deviations are a popular and often useful measure of dispersion. To be sure, a standard deviation is merely the most likely deviation from the mean. It also doesn't take into account the shape of the probability distribution function (this is done better using, for example, entropy, which is a more versatile measure of dispersion).
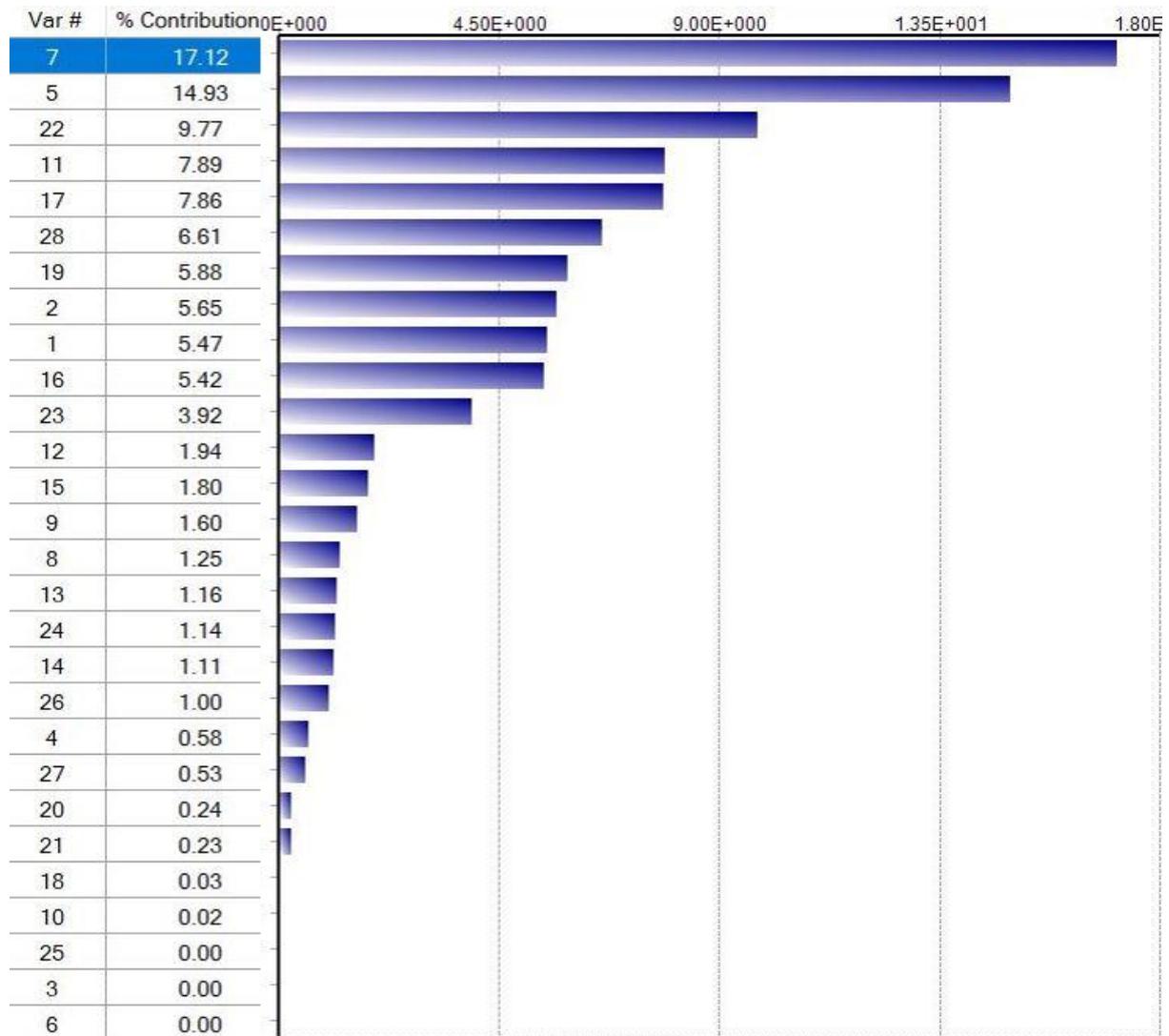
Standard deviations, however, may be 'adjusted' to take into account an interesting aspect of data, namely complexity. Let's see an example. Say you have a portfolio of 28 stocks, all of which are independent (i.e. uncorrelated). In such a case the complexity map of the portfolio is as the one below.

One computes the standard deviation of each stock and may them use it to measure the volatility of the portfolio or other measures of risk. Suppose now that some of the stocks are indeed correlated. Say that the complexity map is now the one below.
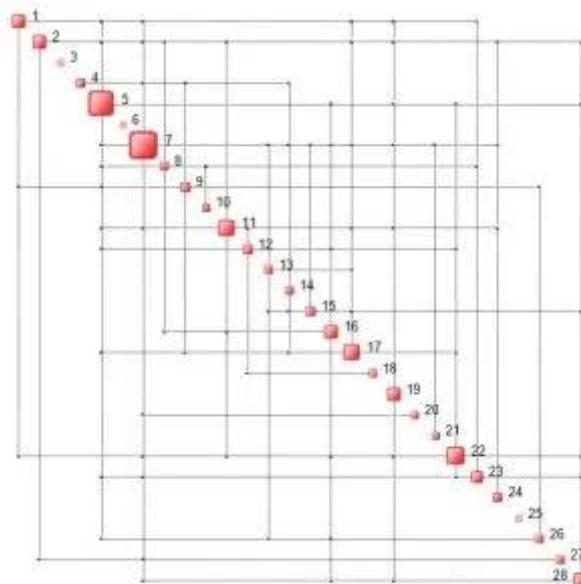
Stocks 5 and 7, for example, are correlated with numerous other stocks, while 3, 6 and 25 are uncorrelated. This is reflected in the Portfolio Complexity Profile (or Portfolio Complexity Spectrum) which ranks the complexity footprint of each stock in the portfolio. This is illustrated below.

Var #	% Contribution
7	17.12
5	14.93
22	9.77
11	7.89
17	7.86
28	6.61
19	5.88
2	5.65
1	5.47
16	5.42
23	3.92
12	1.94
15	1.80
9	1.60
8	1.25
13	1.16
24	1.14
14	1.11
26	1.00
4	0.58
27	0.53
20	0.24
21	0.23
18	0.03
10	0.02
25	0.00
3	0.00
6	0.00

Stock 7 has a footprint of just over 17% while stock 5 is responsible for nearly 15% of the complexity of the portfolio.

Clearly, just like in the previous case, one can calculate the standard deviations of all stocks one by one. However, in the first case all stocks were uncorrelated, here some of them are. These two cases are obviously different, in particular from a structural point of view. The question now is this: why not use the information in the Complexity Profile to 'adjust' standard deviations by adding a correction originating from complexity? Clearly, a stock that is heavily correlated to other stocks in a portfolio could be more 'dangerous' than an uncorrelated one. Evidently, it is the job of covariance to express this:

$$\underline{Covariance(i, j) = Correlation(i, j) \times STD(i) \times STD(j)}$$

But why not take this into account also at standard deviation level? One simple way to accomplish this is the following:

$$\text{Adjusted STD} = (1 + \text{Complexity contribution}) \times \text{STD}$$

Basically, stocks that increase portfolio complexity see their standard deviations corrected (increased) by a complexity-based factor. The (ranked) result is illustrated below.

% Contribution to Total Complexity	Stdev	Adjusted Stdev
0.00	0.59	0.59
0.00	1.00	1.00
0.00	1.03	1.03
0.02	1.06	1.06
0.03	1.07	1.07
0.23	1.08	1.08
0.24	1.13	1.14
0.53	1.16	1.17
0.58	1.27	1.28
1.00	1.29	1.31
1.11	1.31	1.32
1.14	1.36	1.37
1.16	1.39	1.41
1.25	1.46	1.48
1.60	1.57	1.59
1.80	1.77	1.80
1.94	1.79	1.83
3.92	1.80	1.87
5.42	2.07	2.18
5.47	2.08	2.19
5.65	2.42	2.55
5.88	2.42	2.56
6.61	2.84	3.02
7.86	2.92	3.14
7.89	2.99	3.22
9.77	3.25	3.57
14.93	3.51	4.03
17.12	3.81	4.46

The bar chart below shows the complexity-induced corrections of standard deviations.

Standard Deviations
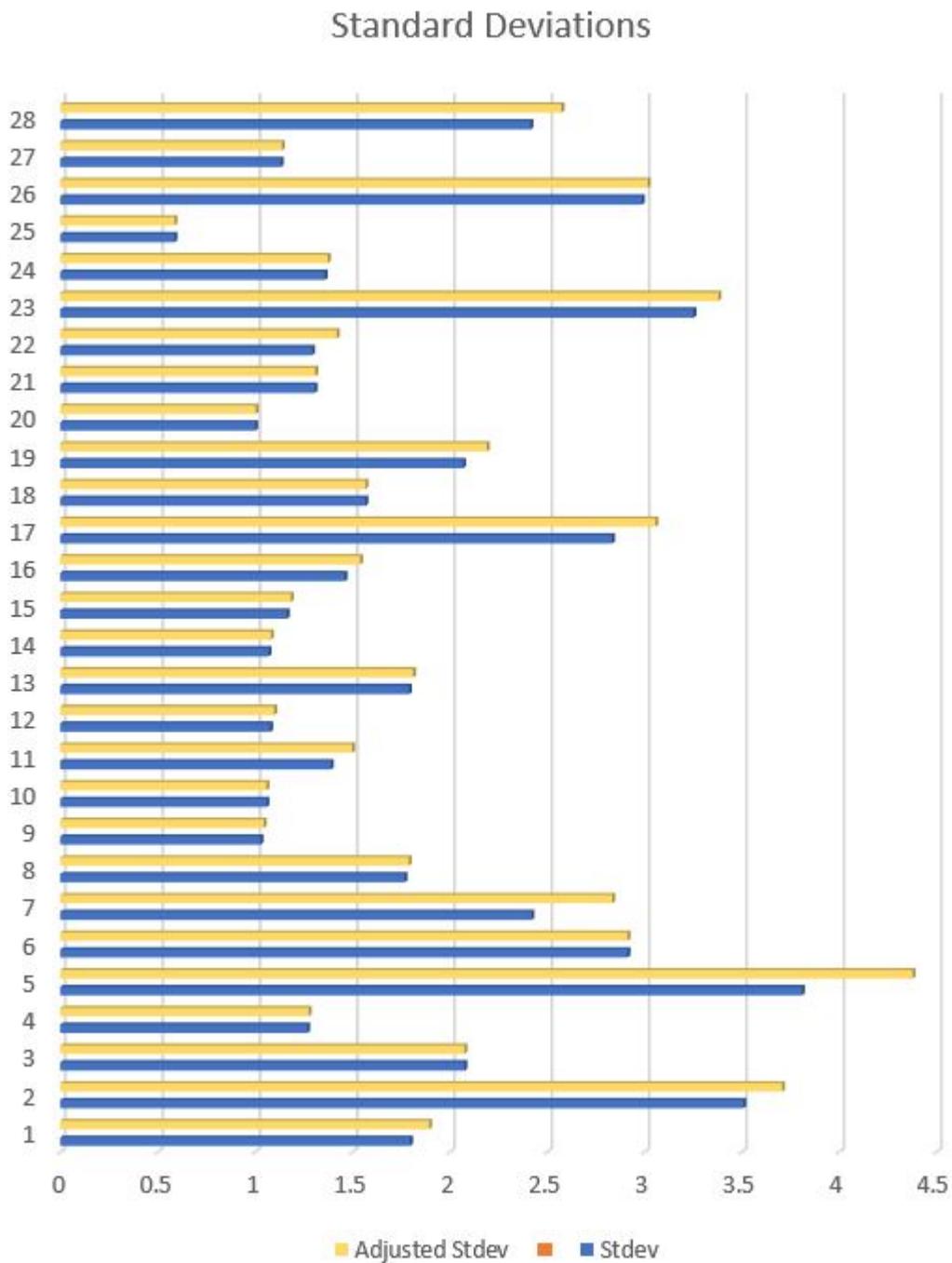
For example, the standard deviation of the biggest complexity contributor – stock 7 – which is 3.81, is incremented by 17.1% (its complexity footprint) to yield a value of 4.46. The norm of the original covariance matrix is 58.21, while the 'corrected' covariance matrix has a norm of 68.15.

Portfolio complexity, which is a factor that is neglected while analysing or designing a portfolio (a covariance matrix is a poor substitute) 'increases' standard deviations, illustrating eloquently the concept of complexity-induced risk.

Doing classical stats may produce overly optimistic results if complexity is neglected. In reality, every system has some degree of complexity, which is invisible to conventional analytics. In reality, there is often more risk than one may think.

Optimum Complexity Ltd.
1 Mayfair Pl, W1J 8AJ
London, United Kingdom

E-mail: dc@optimumcomplexity.com

Website: www.optimumcomplexity.com

Phone: +44 7808 763 348

Disclaimer

The Adviser Optimum Complexity Ltd. is an appointed representative of Atlantide Asset Management Limited which is authorised and regulated by the Financial Conduct Authority

The concepts and methods presented in this document are for illustrative purposes only and are not intended to be exhaustive. This document is not intended as a recommendation, an offer or solicitation for the purchase or sale of any securities or other financial instruments. It is not intended to serve as the basis of any future investment decision and should not be considered as a recommendation or investment advice. No investment decision should be made on the basis of this document in whole or in part. No warranty is made as to the accuracy or reliability of any estimates, opinions, conclusions, recommendations (which may change without notice whether in writing or otherwise) or other information contained in this document and, to the maximum extent permitted by law, all liability and responsibility for any direct or indirect loss or damage which may be suffered by any recipient through relying on anything contained in or omitted from this document is disclaimed without reservation. This document has been published by Optimum Complexity Ltd. based on information provided to it. Optimum Complexity Ltd. has taken care in the preparation of this document to ensure that it accurately reflects the current state of the development of the project, though no representation or warranty is provided as to its accuracy. Certain information contained in this presentation has been obtained from public sources prepared by other parties. No responsibility is assumed for the accuracy or completeness of any information in this document. Any reproduction or distribution of this document, in whole or in part, without the prior written consent of Optimum Complexity Ltd. is prohibited. Reverse-engineering of the concepts, methods or ideas contained in this document is strictly forbidden. The methods described in the present document are protected by US patents. This document should not be accessed by any person in any jurisdiction where it is unlawful to do so. This document is for the intended recipient only and is provided on the condition that it be kept confidential and that it is not copied or circulated in whole or in part.